USN | | | | | | | | | | | BMATM101

First Semester B.E./B.Tech. Degree Examination, June/July 2024
Mathematics – I for Mechanical Engineering Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Find the angle of intersection of the curves $r = a(1 + \cos\theta)$, $r = b(1 - \cos\theta)$	06	L2	CO1
	b.	With the usual notations prove that for the given curve $r = f(\theta)$, $\dfrac{1}{p^2} = \dfrac{1}{r^2} + \dfrac{1}{r^4}\left(\dfrac{dr}{d\theta}\right)^2$.	07	L2	CO1
	c.	Find the radius of curvature of the curve $x^4 + y^4 = 2$ at $(1, 1)$	07	L3	CO1
		OR			
Q.2	a.	Find the pedal equation for the curve $a^m = r^m \cos m\theta$.	08	L2	CO1
	b.	Find the radius of curvature for the curves $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$	07	L3	CO1
	c.	Using modern mathematical tool write a program/code to plot the curve $r = 2 \mid \cos 2\theta \mid$	05	L3	CO5
		Module – 2			
Q.3	a.	Using Maclaurin's theorem prove that $\sqrt{1 + \sin x} = 1 + x - \dfrac{x^2}{2} - \dfrac{x^3}{6} + \dfrac{x^4}{24} - \ldots\ldots$	06	L2	CO2
	b.	If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then find $6U_x + 4U_y + 3U_z = 0$	07	L2	CO2
	c.	Examine the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme value.	07	L3	CO2
		OR			
Q.4	a.	If $u = x + 3y^2 - z^3$, $v = x^2yz$, $w = 2x^2 - xy$, evaluate $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$	08	L3	CO2
	b.	If $u = x^3y^2 + x^2y^3$ where $x = at^2$, $y = 2at$ find $\dfrac{du}{dt}$	07	L3	CO2
	c.	Using modern mathematical tool write a program/code to evaluate $\lim\limits_{x \to \infty}\left(1 + \dfrac{1}{x}\right)^x$	05	L3	CO5
		Module – 3			
Q.5	a.	Solve $\dfrac{dy}{dx} + \dfrac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$	06	L2	CO3
	b.	Find the orthogonal trajectories of cardiod $r = a(1 + \cos\theta)$	07	L2	CO3
	c.	Solve $yp^2 + (x - y)\, p - x = 0$	07	L2	CO3

		OR			
Q.6	a.	Solve $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2 x$	06	L2	CO3
	b.	A body is originally at 80°C and cools down at 60°C in 20 minutes. If the temperature of the air is 40°C, find the temperature of the body after 40 minutes.	07	L3	CO3
	c.	Find the general and singular solution of $xp^2 + xp - yp + 1 - y = 0$	07	L2	CO3
		Module – 4			
Q.7	a.	Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$	06	L2	CO3
	b.	Solve $(D^2 - 6D + 9)y = 6e^{3x} - \log 2$	07	L3	CO3
	c.	Solve $(1+x)^2 \dfrac{d^2 y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = \sin 2[\log(1+x)]$	07	L2	CO3
		OR			
Q.8	a.	Solve $(D^2 - 4D + 13)y = \cos 2x$	06	L2	CO3
	b.	Solve by variation of parameters $(D^2 + 1)y = \tan x$	07	L2	CO3
	c.	Solve $x^2 \dfrac{d^2 y}{dx^2} - 3x\dfrac{dy}{dx} + 4y = (1+x)^2$	07	L2	CO3
		Module – 5			
Q.9	a.	Find the rank of the matrix $\begin{bmatrix} 21 & 22 & 23 & 24 \\ 22 & 23 & 24 & 25 \\ 23 & 24 & 25 & 26 \\ 24 & 25 & 26 & 27 \end{bmatrix}$	06	L2	CO4
	b.	For what value of λ and μ the system of equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ has i) no solution ii) a unique solution iii) infinite number of solutions.	07	L2	CO4
	c.	Using Rayleigh's power method find the dominant eigen value and the corresponding eigen vector of $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by taking $[1, 0, 0]^T$ as the initial eigen vector. (Carry out 6 iterations).	07	L3	CO4
		OR			
Q.10	a.	Solve the system of equation by Gauss Elimination method. $2x + y + 4z = 12$, $4x + 11y - z = 33$, $x + 2y + 5z = 20$	07	L3	CO4
	b.	Solve the system of equations using Gauss-Seidel method by taking (0,0,0) as an initial approximate root. $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$	08	L3	CO4
	c.	Using modern mathematical tool write a program/code to find the largest eigen value of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ by power method.	05	L3	CO5

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